

Weak lensing of the Sunyaev-Zel'dovich sky

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ABSTRACT

We address the question of whether the angular power spectrum of the thermal Sunyaev-Zel'dovich (SZ) sky is further distorted by weak gravitational lensing of foreground large-scale structures. Using an analytic approach to both gaseous and dark halo models, we show that the contamination of weak lensing in the measurement of SZ power is negligibly small, and relative corrections $|\Delta C_l|/C_l$ are less than 3% up to $l = 10^5$. This arises from both the weaker gravitational potentials of low-redshift matter inhomogeneities that can act as lenses for SZ sources (clusters) and the shallower shape of intrinsic SZ power spectrum at large l , in contrast to the cosmic microwave background which can be significantly affected by weak lensing because of the distant location and significant damping of its intrinsic power spectrum at small angular scales.

Key words: cosmic microwave background — galaxies: clusters: general — gravitational lensing — intergalactic medium — large-scale structure of universe

1 INTRODUCTION

Gravitational lensing by foreground large-scale structures of the universe distorts many varieties of background sources. Among these, detection of weak lensing-induced signatures in the cosmic microwave background (CMB) will provide a sensitive probe of matter distribution on large-scales and at high redshifts up to $z \approx 1100$ (e.g. Zaldarriaga & Seljak 1999). On the other hand, the CMB photons can also be distorted by non-gravitational effect along the path from the last scattering surface to us. The most significant pattern is the excess of the CMB power spectrum relative to primordial CMB anisotropy at $\ell > 2000$ (Dawson et al. 2001; Bond et al. 2003), which is produced by the inverse Compton scattering of the CMB photons by intervening energetic electrons confined within massive clusters, known as the Sunyaev-Zel'dovich (SZ) effect (Sunyaev & Zeldovich 1972). Studies of both the SZ-selected clusters and the SZ power spectrum yield important information about the distribution and evolution of hot intracluster medium in the universe.

At scales below 10 arcminutes characterized by $\ell \approx 2000\text{--}10^4$, the CMB power spectrum is dominated by the thermal SZ effect of clusters distributed at intermediate/high redshifts up to $z \approx 1$ (Cooray 2000; Cooray & Hu 2000). An interesting question thus arises: Can the SZ sky be further distorted by the weak gravitational lensing effect of foreground matter inhomogeneities? Namely, we would like to address the question of to what extent the SZ power spectrum at small angular scales should be corrected for because of weak lensing by large-scale structures. It has been noted that the deep gravitational potentials of clusters can lead to both steplike wiggles on the CMB sky (Seljak & Zaldarriaga 2000) and a systematic magnification bias of unresolved background radios behind clusters (Loeb & Refregier 1997). The latter may contaminate the measurement of the thermal SZ effect if the removal of background radio sources is made without inclusion of the inevitable lensing magnification bias. Furthermore, the SZ sky is also correlated with the weak lensing map because the two effects are generated by common massive dark halos (Goldberg & Spergel 1999; Cooray & Hu 2000). The problem we explore in this paper is more straightforward, in the sense that the background source is now the SZ map which is associated with each cluster, while the foreground lenses are large-scale structures along the line of sight. Technically, we can use the formalism similar to the calculation of weak lensing of the CMB power spectrum, which has been well developed in the literature (Seljak 1996; Bernardeau 1997; Zaldarriaga & Seljak 1999; Hu 2000; Zaldarriaga 2000), except that the distribution function of background sources (clusters) should now be taken into account for our case. Throughout this paper we assume a flat cosmological model (Λ CDM) with the parameters determined recently by *WMAP*: $\Omega_M = 0.27$, $\Omega_\Lambda = 0.73$, $\Omega_b h^2 = 0.0224$, $h = 0.71$, $n_s = 0.93$ and $\sigma_8 = 0.84$.

2 FORMALISM

Following the treatment of weak lensing of the CMB under the flat-sky approximation (e.g. Bernardeau 1997; Zaldarriaga & Seljak 1999; Hu 2000; Zaldarriaga 2000), we calculate the lensed SZ temperature fluctuation $T(\boldsymbol{\theta})$ in the direction $\boldsymbol{\theta}$ by

$$T(\boldsymbol{\theta}) = T^s(\boldsymbol{\theta} + \nabla\phi) = \sum_{i=0}^{\infty} \frac{1}{i!} [\nabla\phi(\boldsymbol{\theta}) \cdot \nabla]^i T^s(\boldsymbol{\theta}), \quad (1)$$

where $T^s(\boldsymbol{\theta})$ is the unlensed SZ temperature pattern in direction $\boldsymbol{\theta}$ produced by clusters at redshift z_s , $\nabla\phi$ denotes the lensing deflection angle by foreground large-scale structures along the line of sight, and the projected potential ϕ is related to the three-dimensional potential $\Phi(\mathbf{r})$ of density perturbation $\delta(\mathbf{r})$ at z through

$$\nabla\phi = \frac{2}{c^2} \int_0^{z_s} \left(\frac{D_{\text{ds}}}{D_s} \right) \nabla_{\perp} \Phi dr; \quad (2)$$

$$\nabla_{\perp}^2 \Phi = \frac{3}{2} \Omega_M H_0^2 \frac{\delta(\mathbf{r})}{a}, \quad (3)$$

in which D_{ds} and D_s are the angular diameter distances from the matter inhomogeneities at z and the observer at $z = 0$ to the sources at z_s , respectively, r is the comoving distance and $a^{-1} = (1+z)$. The power spectrum of the lensed SZ temperature map up to the fourth order in deflection angle in terms of equation (1) is thus (e.g. Cooray 2003)

$$\begin{aligned} C_l = & C_l^s - C_l^s \int \frac{d^2 \mathbf{l}'}{(2\pi)^2} C_{l'}^{\phi} \frac{(\mathbf{l} \cdot \mathbf{l}')^2}{|\mathbf{l}'|^4} + \int \frac{d^2 \mathbf{l}'}{(2\pi)^2} C_{l'}^{\phi} C_{|1-\mathbf{l}'|}^s \frac{[(1-\mathbf{l}') \cdot \mathbf{l}']^2}{|\mathbf{l}'|^4} \\ & + \frac{1}{2} C_l^s \left[\int \frac{d^2 \mathbf{l}'}{(2\pi)^2} C_{l'}^{\phi} \frac{(\mathbf{l} \cdot \mathbf{l}')^2}{|\mathbf{l}'|^4} \right]^2 - \int \frac{d^2 \mathbf{l}'}{(2\pi)^2} \frac{d^2 \mathbf{l}''}{(2\pi)^2} C_{|1-\mathbf{l}'|}^s C_{l'}^{\phi} C_{l''}^{\phi} \frac{[(1-\mathbf{l}') \cdot \mathbf{l}']^2}{|\mathbf{l}'|^4} \frac{[(1-\mathbf{l}') \cdot \mathbf{l}'']^2}{|\mathbf{l}''|^4} \\ & + \frac{1}{2} \int \frac{d^2 \mathbf{l}'}{(2\pi)^2} \frac{d^2 \mathbf{l}''}{(2\pi)^2} C_{|1-\mathbf{l}'-\mathbf{l}''|}^s C_{l'}^{\phi} C_{l''}^{\phi} \frac{[(1-\mathbf{l}'-\mathbf{l}'') \cdot \mathbf{l}']^2}{|\mathbf{l}'|^4} \frac{[(1-\mathbf{l}'-\mathbf{l}'') \cdot \mathbf{l}']^2}{|\mathbf{l}''|^4} \end{aligned} \quad (4)$$

where C_l^s is the unlensed power spectrum of the SZ map, and C_l^{ϕ} corresponds to the lensing power spectrum defined as

$$C_l^{\phi} = \frac{9\Omega_M^2 H_0^4}{c^4} \int_0^{z_{\text{dec}}} w(z_s) dz_s \int_0^{z_s} \left(\frac{D_{\text{ds}}}{D_s} \right)^2 \frac{dr}{a^2} P^{\text{m}} \left(\frac{l}{D_A}, z \right), \quad (5)$$

$z_{\text{dec}} \approx 1100$ is the decoupling redshift, D_A is the comoving angular diameter distance, $P^{\text{m}}(k, z)$ is the matter power spectrum at z , and $w(z_s)$ is the distribution function of the SZ sources (i.e. clusters). If the mass function of dark halos is $d^2 n/dM dV$, then $w(z_s)$ can be expressed as

$$w(z_s) = \frac{\frac{d^2 V}{dz_s d\Omega} \int_{M_{\text{min}}}^{\infty} dM \frac{d^2 n(M, z_s)}{dM dV}}{\int_0^{z_s} \frac{d^2 V}{dz_s d\Omega} dz_s \int_{M_{\text{min}}}^{\infty} dM \frac{d^2 n(M, z_s)}{dM dV}}, \quad (6)$$

where $d^2 V/dz_s d\Omega$ denotes the comoving volume per unit redshift and per steradian. We have also tested the observationally determined X-ray luminosity function of clusters (Rosati, Borgani & Norman 2002) for $L_X > 10^{43} \text{ erg s}^{-1}$ instead of the mass function $d^2 n/dM dV$ and found that the modification is only minor. In principle, the lower mass limit M_{min} can be taken to be 0 because low-mass halos (e.g. $M < 10^{13} M_{\odot}$) almost make no contributions to SZ effect unless the self-similar model is adopted for gas distribution. Nevertheless, inclusion of the low-mass halos may significantly alter the distribution function $w(z_s)$. So, we will take $M_{\text{min}} = 10^{14} M_{\odot}$ in the following calculation, which corresponds roughly to a lower luminosity cutoff of $L_X > 10^{43} \text{ erg s}^{-1}$. Such a choice of the lower limit is also consistent with the prediction of preheating model, in which the preheated baryons cannot be trapped in the shallower gravitational potential wells of low-mass halos (see below).

The undistorted thermal SZ power spectrum C_l^s is composed of the so-called Poisson term C_l^P and clustering term C_l^C , respectively,

$$C_l^P = g^2(x) \int_0^{z_{\text{dec}}} dz_s \frac{d^2 V}{dz_s d\Omega} \int_{M_{\text{min}}}^{\infty} dM \frac{d^2 n(M, z_s)}{dM dV} |y_l(M, z_s)|^2; \quad (7)$$

$$C_l^C = g^2(x) \int_0^{z_{\text{dec}}} dz_s \frac{d^2 V}{dz_s d\Omega} P^{\text{lin}} \left(\frac{l}{D_0}, z_s \right) \left[\int_{M_{\text{min}}}^{\infty} dM \frac{d^2 n(M, z_s)}{dM dV} b(M, z_s) |y_l(M, z_s)| \right]^2, \quad (8)$$

where $y_l(M, z_s)$ is the Fourier transform of the Compton y -parameter of the cluster of mass M at z_s , $D_0 = (1+z_s)D_s$ is the comoving distance, $P^{\text{lin}}(k, z_s)$ is the linear matter power spectrum at z_s , and $b(M, z_s)$ is the bias parameter, for which we use the analytic prescription of (Mo & White 1996). The frequency dependence term $g(x)$ is defined as $g(x) = (x^2 e^x)[4 - x \coth(x/2)]/(e^x - 1)^2$, in which $x = h\nu/kT_{\text{CMB}}$ is the dimensionless frequency, and T_{CMB} is the temperature of the present CMB. The observing frequency will be fixed at $\nu = 30 \text{ GHz}$ in this study.

Mass function of massive dark halos is assumed to follow the standard Press-Schechter function

$$\frac{d^2 n(M, z)}{dM dV} = -\sqrt{\frac{2}{\pi}} \frac{\bar{\rho}}{M} \frac{\delta_c}{\sigma^2} \frac{d\sigma}{dM} \exp\left(-\frac{\delta_c^2}{2\sigma^2}\right), \quad (9)$$

in which σ is the linear theory variance of the mass density fluctuation in sphere of mass M : $\sigma^2 = (1/2\pi^2) \int_0^\infty k^2 P^{\text{lin}}(k) |W(kR)|^2 dk$, $W(kR)$ is the Fourier representation of the window function, and $W(kR) = 3([\sin(kR) - kR \cos(kR)]/(kR)^3$. We adopt the present-day linear matter power spectrum $P^{\text{lin}}(k)$ given by Bardeen et al. (1986) and a power-law form ($\propto k^{n_s}$) for the primordial matter density fluctuation.

At small scales below $\sim 10'$, matter power spectrum is strongly affected by non-linear structures. Following Seljak (2000) and Cooray, Hu & Miralda-Escudé (2000) we use a halo approach to calculate the non-linear dark matter power spectrum $P^{\text{m}}(k, z)$, which consists of a single halo term $P^{\text{1h}}(k, z)$ and a clustering term $P^{\text{2h}}(k, z)$:

$$\begin{aligned} P^{\text{1h}}(k, z) &= \int dM \frac{d^2 n(M, z)}{dM dV} \left| \frac{\rho_h(M, z, k)}{\bar{\rho}(z)} \right|^2 \\ P^{\text{2h}}(k, z) &= P^{\text{lin}}(k, z) \left[\int dM \frac{d^2 n(M, z)}{dM dV} b(M, z) \frac{\rho_h(M, z, k)}{\bar{\rho}(z)} \right]^2, \end{aligned} \quad (10)$$

where $\rho_h(M, z, k)$ is the Fourier transform of dark matter density profile $\rho_h(M, z)$ of a halo M at z , and $\bar{\rho}(z)$ is the mean mass density of the background universe at z . We adopt the functional form suggested by numerical simulations (Navarro, Frenk & White 1997; NFW) for $\rho_h(M, z)$ and fix the concentration parameter in terms of the empirical fitting formula of Bullock et al. (2001).

Finally, we need to specify the gas distribution of dark halos in order to evaluate the Compton y -parameter. Since the weak lensing effect of the SZ map may occur at very small angular scales, we should employ a more realistic model for the gas distribution in the central regions of clusters instead of the commonly adopted self-similar model in literature. Recall that the SZ power spectra predicted by the self-similar model and other physically-motivated models such as preheating and radiative cooling become indistinguishable roughly at $l < 2000$. For simplicity, here we work with an analytic preheating model, in which the gas distribution in massive halos can be obtained by combining the equation of hydrostatic equilibrium with a modified entropy profile $S(r) = S_{\text{floor}} + S_{\text{NFW}}(r)$. Namely, a constant entropy floor of $S_{\text{floor}} \approx 120 \text{ keV cm}^2$ is now added to the entropy profile $S_{\text{NFW}}(r)$ predicted by the self-similar model which again takes the universal density profile (NFW). This model allows us to essentially reproduce the observed X-ray surface brightness distribution (i.e. β model) and statistical properties of groups and clusters (e.g. the X-ray Luminosity - temperature relation and entropy distribution) (Babul et al. 2002; Xue & Wu 2003).

3 RESULTS

We first demonstrate in Figure 1 the lensing power spectrum C_l^ϕ for the SZ sky. For comparison, we have also shown the result for the CMB calculated by setting $w(z_s) = \delta(z_s - z_{\text{dec}})$ in eqn.(5) and running the publicly available code CMBFAST. At small scales, the lensing power spectra for the SZ and CMB maps appear to be similar in shape, but the lensing amplitude for the SZ sources is about two orders of magnitude smaller than that for the CMB. This is simply because there are much more intervening matter inhomogeneities acting as lenses for the CMB photons than those for the SZ signals generated by clusters located mostly at $z < 1$. Furthermore, C_l^ϕ peaks at lower l for the SZ map than for the CMB, consistent with the fact that the large-scale structures responsible for weak lensing of the SZ sky are at low redshifts and therefore, subtend relatively large angles. Recall that gravitational lensing conserves the surface brightness of background source. Therefore, C_l^ϕ acts as a narrow window function on the background power spectrum C_l^s via eqn.(4), which alters the intrinsic patterns of C_l^s . Our intuition in terms of this simple argument is that the weak lensing effect on the SZ power spectrum is probably much smaller than the lensing signature on the CMB, if the lensing induced correction is simply proportional to C_l^ϕ . Finally, the lensing power spectra at small scales are dominated by non-linear structures, and linear approximation of matter power spectrum $P^{\text{lin}}(k, z)$ may give rise to an underestimate of the lensing effect.

Nevertheless, the extent to which background sources are affected by weak lensing of foreground large-scale structures also depends critically on the shape of the source power spectrum. We begin by recalling how the CMB can effectively be lensed by large-scale structures (see Hu 2000): At large scales $l < 100$, the CMB power spectrum can be considered to be a slowly varying function of l (see Figure.2), and we can approximately take C_l^s out of the second integral in eqn(4). The lensing induced two terms to the first order approximation in deflection angle cancel if $1 - l'$ is further replaced by 1 . This yields $C_l \approx C_l^s$. At small scales, existence of the strong Silk damping of intrinsic CMB power spectrum at $l > 2000$ makes the contribution of the first correction term in eqn.(4) considerably small, and the second term dominates the lensing correction. As a result, weak lensing always leads to an increase of the CMB power at small angular scales, as shown in Figure 2. Now we turn to the SZ power spectrum: Unlike the intrinsic CMB, there is no significant damping effect on the thermal SZ power spectrum over all angular scales. Rather, the SZ power spectrum has a long, slowly decreasing tail at high l . In other word, the

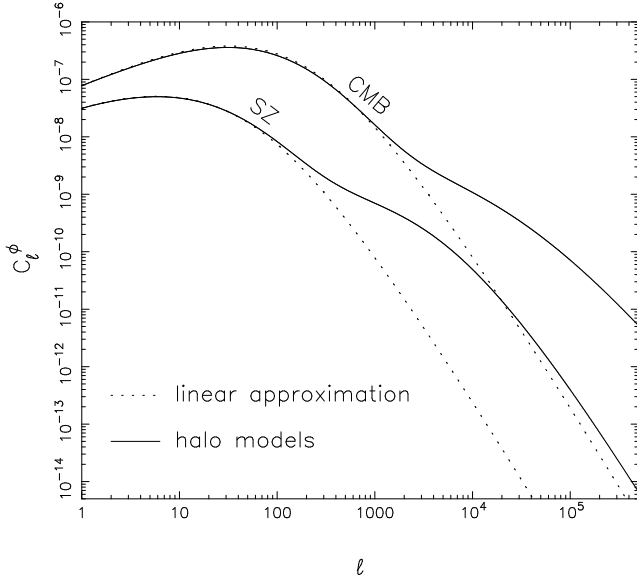


Figure 1. Power spectra of weak lensing for CMB and SZ map. Dotted and solid lines represent the results of linear and non-linear matter power spectra, respectively.

Table 1. Numerical Results

l	100	1000	10000	100000
$\frac{ C_l - C_l^s }{C_l^s} (\%)$	0.0025	0.0065	0.53	2.51
$\frac{C_l(\text{fourth order})}{C_l(\text{second order})}$	7.9×10^{-10}	9.2×10^{-9}	1.5×10^{-5}	6.7×10^{-4}

first negative term in eqn.(4) can no longer be ignored. Applying the approximation that C_l^s is slowly varying and $1 - l' \approx 1$ to eqn.(4), we have $C_l \approx C_l^s$ to the first order in deflection angle. Our numerical calculation reaches essentially the same conclusion. In Table 1 we have listed the relative corrections $|C_l - C_l^s|/C_l^s$ for a set of multipoles from $l = 100$ to $l = 10^5$. It turns out that the lensing induced corrections are well below 3% even up to $l \approx 10^5$. Meanwhile, we have given in Table 1 the power spectrum ratios of the fourth order correction [the last three terms of eqn.(4)] to the second order approximation in C_l^ϕ [the second and third terms of eqn.(4)]. While the contribution of higher order corrections to the power spectrum C_l becomes increasingly important with the decrease of angular scale, the effect of higher order terms on the evaluation of C_l is still negligibly small on scales up to $l = 10^5$ as compared with the contribution from the first order approximation in deflection angle.

4 CONCLUSIONS

It has been shown that the angular power spectrum of the thermal SZ sky is almost unaffected by the weak lensing of foreground large-scale structures. This is primarily due to both the relatively weaker gravitational potentials of low-redshift matter inhomogeneities that can act as lenses for SZ sources and the shallower shape of SZ power spectrum at smaller angular scales. Note that we have adopted the gas density profile predicted by the so-called preheating model, which results in a more rapid decline of the SZ power spectrum at large l than the one predicted by the self-similar model or cooling model (Wu & Xue 2003). In other word, we have already provided an optimistic estimate of the weak lensing effect on the SZ power spectrum since significant fluctuations intrinsic to background sources are more easily be distorted by lensing. This explains why weak lensing of large-scale structures of the universe can effectively alter the CMB sky at $z_s \approx 1100$ as the CMB power spectrum

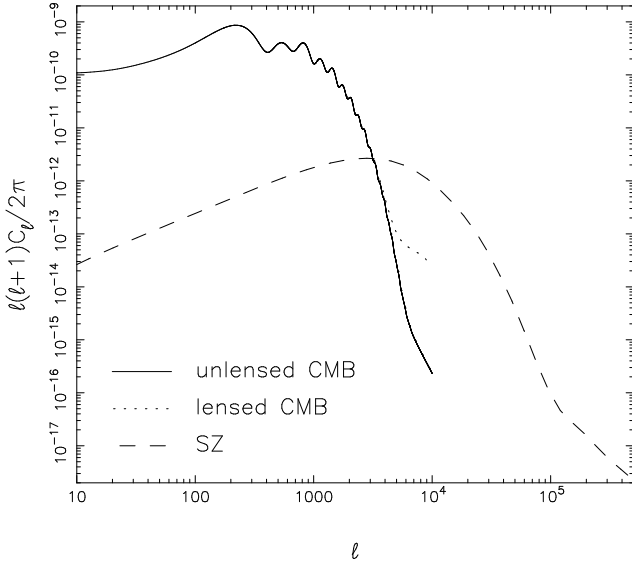


Figure 2. Angular power spectra of the unlensed and lensed CMB and SZ map. The lensed SZ power spectrum is almost identical to the intrinsic one, and the relative difference reaches only 3% at $l \approx 10^5$.

exhibits the sharp damping signatures at high l (Hu 2000). The angular power spectrum of 21cm map of the reionization at $z \sim 10$ also demonstrates a significant decline feature at $l = 10^3 - 10^4$ (e.g. Zaldarriaga, Furlanetto & Hernquist 2003), and it is thus expected that the 21cm fluctuation of the reionization should be affected by weak lensing (Pen 2003). Nonetheless, the present study has eliminated the concern that the SZ sky may be further contaminated by weak lensing of foreground large-scale structures of the universe.

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